Causal Parameters, Structural Equations, Treatment Effects and Randomized Evaluation of Social Programs

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Policy Problems considered by econometricians

1. *Policy evaluation problem and Problem of causal inference*

What are the effects of interventions

1. *Policy forecasting problem*
   a. forecasting the effects of a policy experienced and evaluated in one environment that is proposed for application in a different social and economic environment
   b. forecasting the effects of a new policy never previously experienced.
A Framework For Counterfactuals And Causal Inference:

Models of Isolated Individuals

(A-1) Two potential states, “untreated” and “treated,” with corresponding potential outcomes given by the variables \((y_0, y_1)\),
\[y_0, y_1 \in \mathbb{R}.
\]

(A-2) No market or social interactions among agents in the hypothetical world. (Relaxed below)

(A-3) A static model

Models For Potential Outcomes

\[y_0 = g_0(x)\]
\[y_1 = g_1(x)\]
$\mathcal{D}_i$ domain of function $g_i$

\[ g_0 : \mathcal{D}_0 \mapsto \mathbb{R}, \]

\[ g_1 : \mathcal{D}_1 \mapsto \mathbb{R} \]

\[ \mathcal{D}_i \subseteq \mathbb{R}^J. \]

$x$ are causal variables.

They completely determine the outcomes $y_0, y_1$. 
The arguments in, and functional forms of, $g_0$ and $g_1$ are specified by economic theory. The elements of $x$ are externally specified by a well defined hypothetical variation.

Economic and scientific theory produces causal functions in which the inputs (or externally specified $x$ variables) affect outputs of $y, y_1$. 
Causality is in the mind.

‘It is sometimes said that the laws of economics are ‘hypothetical’. Of course, like every other science, it undertakes to study the effects which will be produced by certain causes, not absolutely, but subject to the condition that other things are equal and that the causes are able to work out their effects undisturbed. Almost every scientific doctrine, when carefully and formally stated, will be found to contain some proviso to the effect that other things are equal; the action of the causes in question is supposed to be isolated; certain effects are attributed to them, but only on the hypothesis that no cause is permitted to enter except those distinctly allowed for” (A. Marshall, 1920, p. 36)
Connecting To A Hypothetical Population of Possible Worlds

\((\Omega, \mathcal{A}, P)\), a probability space. \(\omega \in \Omega\). \(\Omega\) is a set of individuals in a hypothetical super-population

\[(Y_0(\omega), Y_1(\omega))\]

Associated with each potential outcome is a random vector \(X(\omega)\) of \(J\) dimensional explanatory or “causal” variables that are \textit{causes} of \(Y_0(\omega)\) and \(Y_1(\omega)\).

(1a) \[Y_0(\omega) = g_0(X(\omega))\]

(1b) \[Y_1(\omega) = g_1(X(\omega))\]

g_0 and g_1 are measurable functions.
$g_0, g_1$ functions play two roles

1. They describe how the random vector $X(\omega)$ is functionally related to the random variables $Y_0(\omega), Y_1(\omega)$.

2. Specify what values the outcomes would have taken had the causes $X(\omega)$ taken alternative values.

We assume

- Support of $X(\omega)$ lies within the domain of the functions.
- Support of $Y(\omega)$ lies within the range of the functions.
- Support of $X(\omega)$ need not equal the domain of the functions.
Rigorous Definitions of Causal Effects

The causal effect of $x_j$ on $y_i (i = 0, 1)$ is obtained from varying $x_j$. The *causal effect of $x_j$ on $y_i$*, fixing the remaining coordinates of $x$:

$$
(2) \quad [\Delta y_i \mid \Delta x_j = x'_j - x''_j] = g_i(x_1, \ldots, x'_j, \ldots, x_J) - g_i(x_1, \ldots, x''_j, \ldots, x_J)
$$

Requires that the $x_j$ can be independently varied required that these variables be variation-free:

$$(x_1, \ldots, x_J) \in \mathcal{X}_1 \times \ldots \times \mathcal{X}_J.$$  

(More generally: measurable separable)
*Person-specific* causal effect of $x_j$ evaluating equation (2) at the characteristics of person $\omega$ except for the variable being manipulated:

$$g_i(X_1(\omega), \ldots, X_{j-1}(\omega), x_j', X_{j+1}(\omega), \ldots, X_J(\omega))$$

$$-g_i(X_1(\omega), \ldots, X_{j-1}(\omega), x_j, X_{j+1}(\omega), \ldots, X_J(\omega))$$

(3) **Limit Causal Effect of $x_j$ on $y_i$**

$$y_i = \frac{\partial y_i}{\partial x_j} = \frac{\partial g_i(x)}{\partial x_j}.$$
$D(\omega)$ is indicates whether in a hypothetical population person $\omega$
is in regime “1” or not. Thus $D(\omega) = 1$ if $Y_1(\omega)$ is observed and
$D(\omega) = 0$ if $Y_0(\omega)$ is observed.

$D(\omega)$ is measurable $\sigma(Z(\omega))$

(4) $D(\omega) = 1$ if $Z(\omega) \in \tilde{Z}$,

$D(\omega) = 0$ if $Z(\omega) \in Z \setminus \tilde{Z}$.

Deterministic relationship over a more general domain than just the
support of $Z(\omega)$

$$d = 1 \text{ if } z \in \tilde{Z}, \quad d = 0 \text{ if } z \in D_d \setminus \tilde{Z}$$

$D_d$ and $\tilde{Z} = \tilde{Z} \cap Z$. 
Measured outcome, random variable $Y(\omega)$:

(5) \[ Y(\omega) = D(\omega)Y_1(\omega) + (1 - D(\omega))Y_0(\omega). \]

(Switching Model in *Econometrics*, Quand 1972 Roy, 1951)

(6) \textit{Treatment Effect Causal Effect:}

\[ \Delta(x) = y_1(x) - y_0(x) = g_1(x) - g_0(x). \]

This expression evaluated at the characteristics of individual $\omega$ is

\[ \Delta(X(\omega)) = Y_1(\omega) - Y_0(\omega) = g_1(X(\omega)) - g_0(X(\omega)). \]
Causal effect of changing $D$ while holding $X$ constant at $X = x$:

$$h(d(z), y_1(x), y_0(x)) = d(z)y_1(x) + (1 - d(z))y_0(x)$$

and

$$\tilde{h}(z, x) = d(z)y_1(x) + (1 - d(z))y_0(x)$$

$$\Delta(x) = h(1, y_1(x), y_0(x)) - h(0, y_1(x)), y_0(x))$$

**Treatment Effect Causal Effect**

$$\Delta(x) = \tilde{h}(z, x) - \tilde{h}(z', x)$$

for any $z \in \tilde{Z}$ and $z' \in \tilde{Z}^c$
$X$ and $Z$ random variables

$Y_0(\omega), Y_1(\omega)$ and $D(\omega)$ in a hypothetical population are degenerate given the causes, $X(\omega)$ and $Z(\omega)$. 
Producing Nondegenerate Random Variables

Break \( X(\omega) \) and \( Z(\omega) \) into observed \((X_o(\omega), Z_o(\omega))\)
and unobserved components \((X_u(\omega), Z_u(\omega))\).

Suppose Causal Functions Additively Separable:

\[
\begin{align*}
(7a) \quad Y_0(\omega) &= g_{0o}(X_o(\omega)) + g_{0u}(X_u(\omega)) \\
(7b) \quad Y_1(\omega) &= g_{1o}(X_o(\omega)) + g_{1u}(X_u(\omega)).
\end{align*}
\]

“Error Terms”

\[
U_0(\omega) = g_{0u}(X_u(\omega)) \text{ and } U_1(\omega) = g_{1u}(X_u(\omega)).
\]

Assume

\[
\begin{align*}
(8a) \quad E(U_0(\omega) \mid X_o(\omega)) &= 0 \\
(8b) \quad E(U_1(\omega) \mid X_o(\omega)) &= 0
\end{align*}
\]

Mean outcome equations:

\[
\begin{align*}
(9a) \quad E(Y_0(\omega) \mid X_o(\omega)) &= g_{0o}(X_o(\omega)) \\
(9b) \quad E(Y_1(\omega) \mid X_o(\omega)) &= g_{1o}(X_o(\omega)).
\end{align*}
\]
Two points of evaluation of \( X_o(\omega), x_o = (x_{o,1}, ..., x_{o,J}) \) and \( x'_o = (x'_{o,1}, ..., x'_{o,J}) \). Causal effect of a change in \( x_o \) for person \( \omega \) is

\[
\Delta Y_i(\omega) = g_i(x_o, X_u(\omega)) - g_i(x'_o, X_u(\omega)).
\]

Average Causal Effect:

\[
E(Y_i(\omega) \mid X_o(\omega)=x_o) - E(Y_i(\omega) \mid X_o(\omega)=x'_o) = \Delta Y_i(\omega)
\]

\[
= g_{io}(x_o) - g_{io}(x'_o).
\]

Same as Individual Causal Effect:
Failure of (8a) and (8b) gives rise to simultaneous equations bias:

$$E(Y_i(\omega) \mid X_o(\omega)) \neq g_{io}(X_o(\omega)).$$
Assume

(1) That the support of $X(\omega)$, $\mathcal{X}$, is open and connected.

(2) Conditional means of $Y_i$ given $X(\omega)$ are continuous and bounded functions of $X(\omega)$.

Then

(3) There is a point in the support of $X(\omega)$ say $(X_o(\omega), X_u^*(\omega))$ where,
\begin{equation}
\int_{\mathcal{X}_u} E(Y_i(\omega) \mid X_o(\omega), X_u(\omega)) g(X_u(\omega) \mid X_o(\omega)) dX_u(\omega)
= E(Y_i(\omega) \mid X_o(\omega), X^*_u(X_o(\omega))).
\end{equation}

Not a Marshallian causal function because variations in $X_o(\omega)$ shift the implicit point of evaluation of the unobservables $X^*_u(X_o(\omega))$. 
1 Identification: Determining Causal Models From Data Both Hypothetical and Actual

• Consider a model space $M$.

• Class of all models that are considered as worthy of consideration.

• Elements $m \in M$ are possible theoretical models.

Two attributes of a model:

1. a. what one can observe about the model in a given set of data and

   b. what one would like to know about the model. Data may be hypothetical or actual.

Define functions $g : M \rightarrow T$ and $h : M \rightarrow S$ where $T$ and $S$ are spaces chosen so that $g(M) = T$ and $h(M) = S$.

$S$ is the source or data space and $T$ is the target space.

$m \in M$, $h(m) = s \in S$ represents those characteristics
of the model that can be observed in the available data. Map 

\[ g(m) = t \in T \]

applied to the model gives the characteristics of a model that we would like to identify.

Some parameters may be of interest while others are not, and models may only be partially identified.

Only if one is interested in determining the entire model would \( T = M \) and then \( g \) would be an identity map.

Many models \( m \) may be consistent with the same source space. \( h \) is not necessarily one to one.

Identification problem: is to determine whether elements in \( T \) can be uniquely determined from elements in \( S \).

\[ f \equiv g \circ h^{-1}. \]

The identification problem arises because for some \( s \in S \), \( f(s) \) may have more than one element in \( T \).
In that case more than one interpretation of the same evidence is available. If we limit attention to $T$ via $g$, rather than $M$, $f$ is more likely to map points of $S$ into points of $M$. The goal of identification analysis is to find restrictions, $R$, to modify $f$ to $f^R$ so that $f^R(s)$ has at most one element in $T$, i.e. so that only one story is possible given the data.

In the usual form of the identification problem, restrictions are imposed on the admissible models in $M$ so $R \subseteq M$.

For each $s \in S$ define $f^R(s) = g(h^{-1}(s) \cap R)$.

If $R$ is chosen so that for all $s \in S$, $f^R(s)$ has at most one element in $T$, $R$ forms a set of identifying restrictions.

Thus $R \subseteq M$ identifies $g$ from $h$ when for any $s \in S$, $f^R(s) = g(h^{-1}(s) \cap R)$ contains at most one element in $T$. 

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If $R$ is too restrictive, for some values of $s$, $f^R(s) = \emptyset$, the empty set, so $R$ may be incompatible with some $s$, i.e. some features of the model augmented by $R$ are inconsistent with some data. In this case, $R$ forms a set of over-identifying restrictions.

Otherwise, if $f^R(s)$ contains exactly one element in $T$ for each $s \in S$, $R$ forms a set of just-identifying restrictions.

Often not possible to observe the same person (i.e., the same $\omega$) simultaneously in both the $D(\omega) = 1$ and the $D(\omega) = 0$ state.


Sufficient to observe two individuals with the same values of $X$ but different values of $D$, if $\omega, \omega'$

\[ X(\omega) = X(\omega'). \] (Panel Data Methods, etc.)
Scientific Method: Rejected in the New Literature on Causal Inference.
2 Non-recursive Models of Causality

$Y$ and some or all of the components of $X$ can be jointly determined or interrelated. Imposes severe restrictions on the causal parameters that can be defined in such models because it restricts the possibilities of variation in the causes.
Paradigm for this analysis a model of market demand and supply: (drop $\omega$)).

\begin{align*}
(11a) \quad Q^D &= Q^D(P^D, Z^D, U^D) \quad \text{Demand} \\
(11b) \quad Q^S &= Q^S(P^S, Z^S, U^S) \quad \text{Supply}
\end{align*}

$Q^D$ and $Q^S$ are vectors of goods demanded and supplied at prices $P^D$ and $P^S$.

$Z^D$, $Z^S$, $U^D$ and $U^S$ are shifters of market demand and supply equations \((i.e.\text{ determinants of demand and supply})\). Determined outside of the markets: \textbf{external variables}.

$P$ and $Q$ \textbf{internal variables}. 


Marshall’s model of industry equilibrium, (11a) is the demand for a good by a representative consumer while (11b) is the supply function of the representative price-taking firm. Assume that \( Q^D \) and \( Q^S \) are single-valued functions. Equilibrium exists if, \( Q = Q^D = Q^S \) and \( P = P^D = P^S \).

If \((P, Q)\) is uniquely determined as a function of \( Z \) and \( U \), the model is said to be “complete”. Can hypothetically vary \( P^D \) and \( P^S \) to obtain causal effects for (11a) and (11b) as partial derivatives or as finite differences of prices holding other factors constant. Definition of a causal parameter does not require any statement about what is actually observed or what can be identified from data.

(11a) is the conditional expectation of \( Q^D \) given \( P^D, Z^D \) and \( U^D \), (11b) is the conditional expectation of \( Q^S \) given \( P^S, Z^S, U^S \)
\[ E(Q^D \mid P^D, Z^D, U^D) \neq E(Q^D \mid P^D, Z^D). \]

Restriction. (11a) and (11b) have at least one solution

\[ P = P^D = P^S \quad \text{and} \quad Q = Q^D = Q^S, \]

so there is at least one market equilibrium.

In equilibrium, the price of good \( j \) cannot be changed unless the exogenous or forcing variables, \( Z^D, Z^S, U^S, U^D \), are changed.

Under completeness, we can derive the reduced forms:

\[ P = P(Z^D, Z^S, U^D, U^S) \]

\[ Q = Q(Z^D, Z^S, U^D, U^S). \]
Assume that within the model, $Z^D$ and $Z^S$ can be independently varied for each given value of $U^D$ and $U^S$ (i.e. it is possible to vary $Z^D$ and $Z^S$ within the model holding $U^D$ and $U^S$ fixed).

Assuming that some components of $Z^D$ do not appear in $Z^S$, that some components of $Z^S$ do not appear in $Z^D$, and that those components have a nonzero impact on price, one can use the variation in the excluded variables to vary the $P^D$ or $P^S$ in equations (11a) and (11b) while holding the other arguments of those equations fixed. Assume differentiable functions, and let $Z^S_e$ be a variable excluded from $Z^D$.

\[
\frac{\partial Q^D}{\partial P^D} = \left( \frac{\partial Q}{\partial Z^S_e} \right) \bigg/ \left( \frac{\partial P}{\partial Z^S_e} \right)
\]

\[
\frac{\partial Q^S}{\partial P^S} = \left( \frac{\partial Q}{\partial Z^D_e} \right) \bigg/ \left( \frac{\partial P}{\partial Z^D_e} \right).
\]

Under these conditions, we can recover the price derivatives of
(11a) and (11b) even though an equilibrium restriction connects $P^D = P^S$.

Crucial notion in defining the causal parameter for price variation, when the market outcomes are characterized by an equilibrium relationship, is variation in external variables that affect causes (the $P^D$ and $P^S$, respectively, in these examples) but that do not affect causal relationships (i.e. that are excluded from the relationship in question). If an external variable is excluded from the causal relationship so it does not directly affect the causal relationship, the causal law is said to be invariant with respect to variations in that external variable. If the variable in question is a policy variable, the causal relationship is said to be “policy invariant.”
3 Reformulating the Evaluation Problem to the Population Level

\[ 0 < Pr(D(\omega) = 1 \mid X_o(\omega)) < 1 \]

(A-4) \[ 0 < Pr(D(\omega) = 1 \mid X_o(\omega)) < 1 \]

Average Treatment Effect (ATE):

(12a) \[ ATE(X_o(\omega)=x) = E(Y_1(\omega)-Y_0(\omega) \mid X_o(\omega)=x). \]

Treatment on the Treated (TT):

(12b) \[ TT(X_o(\omega)=x) = E(Y_1(\omega)-Y_0(\omega) \mid X_o(\omega)=x, D(\omega)=1). \]

Treatment on the Untreated:

(12c) \[ TUT(X_o(\omega)=x) = E(Y_1(\omega)-Y_0(\omega) \mid X_o(\omega)=x, D(\omega)=0). \]
Sufficient no-feedback condition:

\[(A-5) \quad X_{o,1}(\omega) = X_{o,0}(\omega) \quad a.e.\]
Mean finite change in

\[ Y(w) = D(Z(\omega))Y_1(X(\omega)) + (1 - D(Z(\omega)))Y_0(X(\omega)) \]

with respect to the finite change in the jth coordinate of \( Z_0(\omega) \),

\( Z_j(\omega) : \)

\[
(13) \quad E \left[ \frac{\Delta Y(\omega)}{\Delta Z_j(\omega)} \right] \bigg| X_0(\omega) = x, Z_0(\omega) = z, \Delta Z_j(\omega) \neq 0 \).
\]

\[
(14) \quad MTE (X_0(\omega) = x, Z_0(\omega) = z) = \lim_{\Delta Z_j(\omega) \to 0} E \left[ \frac{\Delta Y(\omega)}{\Delta Z_j(\omega)} \right] \bigg| X_0(\omega) = x, Z_0(\omega) = z.
\]
4 Relationships Among Population Treatment Effects Causal Parameters, Marshallian Causal Functions and Structural Equation Models

*Structural equations* are low dimensional parameterizations of the Marshallian causal functions.

\[(15a)\quad Y_0 = g_0(X(\omega)) = f_0(X_o(\omega), X_u(\omega); \theta_0)\]

\[(15b)\quad Y_1 = g_1(X(\omega)) = f_1(X_o(\omega), X_u(\omega); \theta_1)\]

where \(\theta_0\) and \(\theta_1\) are parameters that generate the \(g_0\) and \(g_1\) functions.

Two distinct advantages: (a) reduce the computational burden of determining \(g_0\) and \(g_1\) and (b) they can be used to extrapolate functions fitted on the support of \(X(\omega)\), where \((\theta_0, \theta_1)\) can be identified, to other domains of definition.

Standard linear structural equations case:
(16a) \[ f_0 = X_o(\omega)' \theta_o + \nu_0(\omega) \]

(16b) \[ f_1 = X_o(\omega)' \theta_1 + \nu_1(\omega) \]

Treatment effect parameters that are determined in one population do not apply to another population unless the relationship between the observables and unobservables is the same in both populations.

The requirement for extrapolation is that

\[ EF(g_u(X_u(\omega))) = E_{F^*}(g_u(X_u(\omega))) \]

where \( F \) and \( F^* \) are the conditional distributions of \( X_u(\omega) \) given \( X_o(\omega) \) for the two populations and that the new distribution \( F^* \) does not change the underlying structural relationship between \( Y(\omega) \) and \( (X_o(\omega); X_u(\omega)) \).

**ATE:**

\[ E(Y_1(\omega) - Y_0(\omega) \mid X_o(\omega) = x) = \]

\[ \int [g_1(X_o(\omega) = x, X_u(\omega)) - g_0(X_o(\omega) = x, X_u(\omega))] dF(X_u(\omega) \mid X_o(\omega) = x) \]
**ATE** generalizes to different populations as long as

\[ E_F(U_0(\omega) \mid X_0(\omega) = x) = E_{F^*}(U_0(\omega) \mid X_0(\omega) = x). \]

Conditional on \( X_o(\omega) = x \). Variations in **ATE** across values of \( X_o(\omega) \) do not, in general, hold constant values of \( X_u(\omega) \) unless representation (7) is invoked and \( X_o(\omega) \perp \perp X_u(\omega) \) or least condition 8(a) and 8(b) holds. These variations are not Marshallian causal variations.
5 The Value of Structural Equations in Making Policy Forecasts

\[ Y = \varphi(X_o(\omega), X_u(\omega)) \text{, where } \varphi : \mathcal{D} \mapsto \mathcal{Y}, \mathcal{D} \subseteq \mathbb{R}^J, \mathcal{Y} \subseteq \mathbb{R}. \]

\( \varphi \) is a Marshallian causal function determining outcome \( Y \).

Known only over \( \text{Supp}(X_o(\omega), X_u(\omega)) = \mathcal{X}_o \times \mathcal{X}_u. \)

Mean outcome conditional on \( X_o(\omega) = x : \)
Source Expectation:

\[ E_S(Y(\omega) \mid X_o(\omega)=x) \]

\[ = \int_{X_u} \varphi(X_o(\omega)=x, X_u(\omega))dF_S(X_u(\omega) \mid X_o(\omega)=x) \]

\[ F_S(X_u(\omega) \mid X_o(\omega)) \text{ dist. of } X_u \text{ given } X_0 \text{ in } S \]

Target Expectation:

\[ E_T(Y(\omega) \mid X_o(\omega)=x) \]

\[ = \int_{X_u^T} \varphi(X_0(\omega)=x, X_u(\omega))dF_T(X_u(\omega) \mid X_o(\omega)=x) \]
Assume $\varphi$ is additively separable

$$\varphi = \varphi_o(X_o(\omega)) + \varphi_u(X_u(\omega)),$$

- Structural or Marshallian causal functions are determined.
- The new policy characterized by an invertible mapping from observed random variables to the characteristics associated with the policy: $c(\omega) = q(X(\omega))$. $c(\omega)$ is the set of characteristics associated with the policy and $q, q : R^J \rightarrow R^J$, is a known invertible mapping.
- $X(\omega) = q^{-1}(c(\omega))$ is solved to associate characteristics that in principle can be observed with the policy. This places the characteristics of the new policy on the same footing as those of the old.
- $Supp(q^{-1}(c(\omega))) \subseteq Supp(X(\omega))$. Ensures that the support of the new characteristics mapped into $X(\omega)$ space is contained in the support of $X(\omega)$. If this condition is not met, structural versions of the nonparametric Marshallian causal functions must be used to forecast the effects of the new policy, to extend it beyond the
support of the source population.
• The forecast effect of the policy on $Y$ is

$$Y_c(\omega) = \varphi(q^{-1}(c(\omega))).$$
6 Comparing the Rubin Model with the Econometric Model

The lack of an explicit model of assignment to treatment gives rise to the logical difficulties in defining a causal effect holding $X$ fixed that were discussed in Section 2.
7 What Treatment Effect and Structural Parameters Are Identified From Social Experiments?

A. Treatment Effects vs. Structural Parameters: A Labor Supply Example

\( \varepsilon \) denotes an unobservable.

\[ h = \text{hours of work}, \]

(17) \[ h = h(W, X, \varepsilon). \]

\( h \) is differentiable \((C^1)\).
Proportional wage taxes at rate $t$ make the after tax wage $W(1 - t)$.

Other Versions of $h$

(18) $h = h(W, X) + \varepsilon$

(17') $h = h(W, X, \varepsilon; \theta)$

(18') $h = h(W, X; \theta) + \varepsilon$

(19) $h = \alpha'X + \beta \ln W + \varepsilon$
Following Marschak, distinguish three cases.

1. Tax $t$ has been implemented in the past and we wish to forecast the effects of the tax in the future in a population with the same distribution of ($W, X, \varepsilon$) variables as prevailed when measurements of tax variation were made.

2. Tax $t$ has been implemented in the past but we wish to project the effects of the same tax to a different population of ($W, X, \varepsilon$) variables.

3. Tax has never been implemented and we wish to forecast the effect of a tax either on an initial population used to estimate (17) or on a different population.
\[(20) \ E(h \mid W, X, T = t_j)\]

\[= \int h(W(1-t_j), X, \varepsilon) dG(\varepsilon \mid X, W, T = t_j).\]

For the entire population this function is

\[(20') \ E(h \mid T = t_j)\]

\[= \int h(W(1-t_j), X, \varepsilon) dG(\varepsilon, X, W \mid T = t_j).\]
Case one resembles case two except for one crucial difference. It is necessary to break (20) or (20') into its components and determine $h(W(1 − T), X, \varepsilon)$ separately from $G(\varepsilon, X, W, T)$.

(1) Knowledge of $h(\cdot)$ is needed on the new population.

Exogeneity:

(A-6) \((X, W, T) \independent \varepsilon\)\n
\[G(\varepsilon \mid X, W, T) = G(\varepsilon).\]

\[E(h \mid W, X, T = t_j) = \int h(W(1 − t_j), X, \varepsilon)dG(\varepsilon)\]

$G' \neq G$, $G'$ must somehow be determined.
If support of $W(1 - t) \overset{\text{def}}{=} W^*$ in the target regime is contained in the support of $W$ in the historical regime, and the conditional distributions of $W^*$ and $W$ given $X, \varepsilon$ are the same, and the supports of $(X, \varepsilon)$ are the same in both regimes,

(a) $\text{Support } (W^*)_{\text{target}} \subseteq \text{Support } (W)_{\text{historical}}$

(b) $G(W^* \mid X, \varepsilon)_{\text{target}} = G(W^* \mid X, \varepsilon)_{\text{historical}}$

(c) $\text{Support } (X, \varepsilon)_{\text{target}} = \text{Support } (X, \varepsilon)_{\text{historical}}$
B. Two Different Cases for Social Experiments

Use randomization to identify Marshallian causal functions and structural parameters. Labor supply equation is \( h = h(t, W, X, \varepsilon) \)

\[(T \perp \varepsilon) \parallel (W, X)\]

\[\Pr(T = t \mid W, X, \varepsilon) = \Pr(T = t \mid W, X).\]
\[ E(h \mid T=t, W, X) = \int h(t, W, X, \varepsilon) dG(\varepsilon \mid T=t, W, X) \]
\[ = \int h(t, W, X, \varepsilon) dG(\varepsilon \mid W, X) \]
\[ E(h \mid T = t', W, X) = \int h(t', W, X, \varepsilon) dG(\varepsilon \mid W, X) \]
\[ E(h \mid T = t, W, X) - E(h \mid T = t', W, X) \]
\[ = \int [h(t, W, X, \varepsilon) - h(t', W, X, \varepsilon)] dG(\varepsilon \mid W, X). \]

\( Fc(W, X) \) is distribution of \((W, X)\).
Population average treatment effect for taxes \((t, t')\):

\[
E_{F_c}(h \mid t) - E_{F_c}(h \mid t') = \int [E(h \mid T=t, W, X) - E(h \mid T=t', W, X)]dF_c(W, X),
\]

Separability of \(h\) in \(\varepsilon\) facilitates.

(A-7) \[ h = h(W, T, X) + \varepsilon, \]

(21) \[ E(h \mid W, X, T = t) - E(h \mid W, X, T = t') = h(W, X, t) - h(W, X, t'). \]

\[ h = \alpha_0 + \alpha_1 \ln(W(1 - t)) + \alpha'_2 X + \varepsilon \]

\[ = \alpha_0 + \alpha_1 \ln W + \alpha_1 \ln(1 - t) + \alpha'_2 X + \varepsilon. \]

Under (A-7), (21) becomes

\[ E(h \mid W, t, X) - E(h \mid W, t', X) = \alpha_1 [\ln(1 - t) - \ln(1 - t')] \]

Social experiments only identify treatment terms and terms that interact with treatment.

Terms for \((W, X)\) are not identified.
Consider the additively separable case $h(W, X, t, \varepsilon) = h(W, X, t) + \varepsilon$. Under assumption (A-7) for randomization, and assuming full compliance, we can recover $h(W, X, t) - h(W, X, t')$.

Randomization operates by *Balancing the bias*.

$$E(\varepsilon \mid T = t', W, A) = E(\varepsilon \mid T = t, W, A).$$

These “control functions” (or conditional bias terms). If we seek to project the findings from one experiment

$$\varepsilon \perp \perp (W, X)$$