Assignment, Sorting and Matching Problems: Introduction to the Rest of the Course

Bring firms back into wage determination:

Issues:

(A) how to match workers to firms - trivial in the case of pure efficiency units models. Not so trivial when workers have different efficiency at different firms.

We start our investigation under the assumptions of:

Perfect certainty on both sides. (No private information)

No transactions costs (mobility costs)

1-1 matches with transferrable utility and money (one worker with one firm). Still problem not trivial.
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Gorman-Lancaster is multi-attributed version of this type of theory

1. a. We have an efficiency units model if the identify of the firm irrelevant: (workers equally productive at all firms) - a model of general human capital. Rearrange workers among firms and get no change in output.

b. We get a model with comparative advantage if workers have different advantages in different sectors but assignment of a worker to a sector does not preclude any other worker going there: sectors may be firms or industrial sectors. (Now sorting matters - and a nontrivial labor supply function).

Koopmans-Beckman and Sattinger - Hedonics - Tinbergen

1. a. A model with absolute advantage if placement of a worker in one firm (sector) precludes other workers 1 - 1 match or (1 - many match can be achieved).

Absolute advantage:

a. i. (a). Place A to $\alpha$
   
   (b). Means B can’t go to $\alpha$.
   
   (c). (Not just relative productivity, but who is best determines assignment). Continuous versions - worker and firms have close substitutes
   
   (d). Discrete version (Koopmans–Beckman) - no close
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(d). Discrete version (Koopmans–Beckman) - no close . . .
substitutes. (Raises rent division problem) // solved in (i).

The discrete version requires no notion of comparing any 2 workers (no need for a scale) // no need to compare - (other notions require such a comparison).

Roy - based on notion of measurable skills so does Gorman-Lancaster.

Start with the simplest model.
Sattinger:

Assume we can rank workers and firms by a skill scale: $\ell$ is amount of labor skill, $c$ is amount of capital owned by firm.

$F(\ell, c)$ is output. $\exists$ a uniform production technology. One worker - one firm match $F_\ell > 0, F_c > 0, F_{\ell\ell} < 0, F_{cc} < 0$

no need to make scale restrictions: (can be increasing) homogeneous output of firms, identical technologies.

(3) Let $G(\ell)$ be cdf of $\ell$ in population. Let $K(c)$ be cdf of $c$ in population. Assume both monotone strictly increasing, density has positive support - no mass points.

Let $W(\ell)$ be wage for worker of type $\ell$. Let $\pi(c)$ “profit” firm of type $c$. 
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Let $W(\ell)$ be wage for worker of type $\ell$. Let $\pi(c)$ “profit” firm of type $c$. 
Assume $\frac{\partial^2 F}{\partial \ell \partial c} > 0$ (not critical). Assume wage function exist: something to be proved.

Firm indexed by $c$:

Max $\ell$: $F(\ell, c) - W(\ell)$

$FOC$ $\frac{\partial F}{\partial \ell} = W'(\ell)$

$SOC$ $\frac{\partial^2 F}{\partial \ell^2} - W''(\ell) < 0$

defines demand for worker of type $\ell$ for firm type $c$.

$$\frac{\partial}{\partial \ell} \left( W''(\ell) - \frac{\partial^2 F(\ell, c)}{\partial \ell^2} \right) = \left( \frac{\partial^2 F}{\partial \ell \partial c} \right) \frac{\partial c}{\partial \ell}$$

> > 0, from SOC

\[ \cdot \cdot \cdot \frac{\partial c}{\partial \ell} \text{ ("best firms match with best workers"}) \]
Assume $\frac{\partial^2 F}{\partial \ell \partial c} > 0$ (not critical). Assume wage function exist: something to be proved.

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Max $\ell$ \hspace{1cm} $F(\ell, c) - W(\ell)$

FOC \hspace{1cm} $\frac{\partial F}{\partial \ell} = W'(\ell)$

SOC \hspace{1cm} $\frac{\partial^2 F}{\partial \ell^2} - W''(\ell) < 0$

Differentiate

FOC totally wrt $\ell$

$$W''(\ell) - \frac{\partial^2 F(\ell, c)}{\partial \ell^2} - \frac{\partial^2 F}{\partial \ell \partial c} \frac{\partial c}{\partial \ell} = 0$$

$$\left( W''(\ell) - \frac{\partial^2 F(\ell, c)}{\partial \ell^2} \right) = \left( \frac{\partial^2 F}{\partial \ell \partial c} \right) \frac{\partial c}{\partial \ell} \quad \text{> 0, from SOC}$$

$\therefore \frac{\partial c}{\partial \ell}$ ("best firms match with best workers")
opposite true if we have $\frac{\partial^2 F}{\partial \ell \partial c} < 0$.

Retain $\frac{\partial^2 F}{\partial \ell \partial c} > 0$: Profits residually determined:
\[
\pi(c) = F(\ell(c), c) - w(\ell(c)).
\]

Observe that we have that roles of $\ell$ and $c$ can be reversed (labor hires capital) and labor incomes could be residually determined.

The continuum hypothesis for skills $\implies$ local returns to scale
\[
dF = F_\ell d\ell + F_c dc
\]
∴ we get product exhaustion.

∴ residual claimant gets marginal product, no matter who is claimant.

Now suppose # of workers $(N_\ell)$

# of capitalists $(N_c)$.

Then let $W_R$ be the reserve price of workers (what they could
opposite true if we have $\frac{\partial^2 F}{\partial \ell \partial c} < 0$.

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Then let $W_R$ be the reserve price of workers (what they could ...
get elsewhere). Let $\pi_R$ be reserve price of capitalist. Let $\ell^*$ be the least productive worker (employed).

We need $W(\ell^*) \geq W_R$. If all capital employed,

$\ell^*$ works with $\frac{c}{\text{least productive capitalists}}$ $c \in [\underline{c}, \bar{c}]$.

Assumes $\pi(\underline{c}) \geq \pi_R$. Assuming these reserve wage constraints are satisfied, we have that

$$N_{\ell} \int_{\ell^*}^{\infty} g(\ell) d\ell = N_c \int_{\underline{c}}^{\bar{c}} k(c) dc.$$ 

This defines an implicit equation:

$$\ell = \varphi(c) \text{ (most productive match)}$$

(has inverse from strictly increasing assumption).

Feasibility requires: $\varphi^{-1}(\ell) = c$

$$\pi(\underline{c}) = F(\ell(\underline{c}), \underline{c}) - W(\ell^*) \geq \pi_R$$

if not satisfied we have unemployed capital: (jack up $c^* > \underline{c}$) until
get elsewhere). Let $\pi_R$ be reserve price of capitalist. Let $\ell^*$ be the least productive worker (employed).

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if not satisfied we have unemployed capital: (jack up $c^* > \underline{c}$) until ...
constraint satisfied. Then wage function is given by FOC (using $\varphi$)

$$W'(\ell) = \frac{\partial F}{\partial \ell}(\ell, \varphi^{-1}(\ell))$$

defines hedonic line

$$W''(\ell) = F_{\ell\ell} + F_{\ell c} \frac{\partial c}{\partial \ell}$$

$\therefore$ SOC satisfied. Thus the wage is given by

$$W'(\ell^*) = \frac{\partial F}{\partial \ell}(\ell^*, \varphi^{-1}(\ell^*)).$$

Competitive labor market forces $w(\ell^*) = w_R$

$$W(\ell) = \int_{\ell^*}^{\ell} \frac{\partial F}{\partial x}(x, \varphi^{-1}(x)) dx + W_R.$$  

“hedonic function”

Similarly

$$\pi(c) = \int_{c_*}^{c} \frac{dF}{dZ}(\varphi(Z), Z) dZ.$$  

Under our assumptions (more workers than firms and unemployed worker), rents are assigned to firms. Density of earnings is obtained
constraint satisfied. Then wage function is given by FOC (using $\varphi$)

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Under our assumptions (more workers than firms and unemployed worker), rents are assigned to firms. Density of earnings is obtained ...
from inverting wage function

\[ W(\ell) = \eta(\ell) \quad \eta^{-1}(W) = \ell \]

density of earnings is

\[ g(\eta^{-1}(W)) \frac{d\eta^{-1}(W)}{dW} \]

density of profits likewise. Now, we can operate more generally-
example is fruitful.
Cobb Douglas
\[ F(\ell, c) = \ell^\alpha c^\beta \quad \alpha > 0, \beta > 0 \]

Assume Pareto distribution of endowments:
\[ g(\ell) = d\ell^{-\gamma} \quad \gamma > 2, \quad \ell \geq 1 \]
\[ k(c) = hc^{-\sigma} \quad \sigma > 2, \quad c \geq 1. \]

This ensures finite variances. Obviously \( F_{\ell c} > 0 \).

Equilibrium:
\[ N_c \int_{c(\ell)}^\infty h x^{-\sigma} \, dx = N_\ell \int_{\ell}^\infty d\eta^{-\gamma} \, d\eta \]
\[ c(\ell) = \left[ \frac{N_\ell d (\sigma - 1)}{N_c h (\gamma - 1)} \right] \frac{1}{1 - \sigma} \frac{1 - \gamma}{(\ell)1 - \sigma}. \]

FOC (on wages)
\[ \alpha \ell^{\alpha - 1} c^\beta = W'(\ell) : \text{substitute for } c(\ell) \text{ to reach} \]
\[ \therefore W'(\ell) = \alpha \left[ \frac{N_\ell d (\sigma - 1)}{N_c h (\gamma - 1)} \right] \frac{\beta}{1 - \sigma} \ell^\beta. \]
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\alpha \ell^{\alpha - 1} c^\beta = W'(\ell) : \text{substitute for } c(\ell) \text{ to reach}
\]

\[
\therefore W'(\ell) = \alpha \left[ \frac{N_\ell d (\sigma - 1)}{N_c h (\gamma - 1)} \right] \frac{\beta}{1 - \sigma} \ell^p
\]
\[ P = \frac{(\alpha - 1)(1 - \sigma) + \beta(1 - \gamma)}{1 - \sigma} \]
\[ W(\ell) = \frac{\alpha(1 - \sigma) \left[ \frac{N_{el}d(\sigma - 1)}{N_{ch}(\gamma - 1)} \right]^{\frac{\beta}{1 - \sigma}}}{\alpha(1 - \sigma) + \beta(1 - \gamma)} \]
\[ \cdot (\ell) \left( \frac{\alpha(1 - \sigma) + \beta(1 - \gamma)}{1 - \sigma} \right) + c_1. \]

Obviously \( W(\ell) \uparrow \) as \( \ell \uparrow \). Convexity or concavity hinges on
\[ P \leq 0 \]
\[ P = (\alpha - 1) + \beta \frac{(1 - \gamma)}{1 - \sigma} \]

if \( \alpha + \beta = 1 \) (CRS)

\[ P = \beta \left[ -1 + \frac{1 - \gamma}{1 - \sigma} \right] \]
\[ = \beta \left[ \frac{\sigma - \gamma}{1 - \sigma} \right] = \frac{\gamma - \sigma}{\sigma - 1} \]

If \( \gamma > \sigma \), \( W(\ell) \) is convex in \( \ell \). (More firms out in tail then workers - they get scarcity payment). If \( \beta \uparrow \) (from CRS) reinforces
\[ P = \frac{(\alpha - 1)(1 - \sigma) + \beta(1 - \gamma)}{1 - \sigma} \]

\[ W(\ell) = \frac{\alpha(1 - \sigma) \left[ \frac{N_{\ell} d(\sigma - 1)}{N_{\ell} h(\gamma - 1)} \right]^{\frac{\beta}{1 - \sigma}}}{\alpha(1 - \sigma) + \beta(1 - \gamma)} \]

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effect (renders capital relatively more productive).

Thus if $\gamma = \sigma$ and $\beta + \alpha > 1$ ($\beta$ big enough) has effect on convexity. Increasing returns to scale gives rise to convexity. Now

$$\pi(c) = \ell^\alpha c^\beta - w(\ell)$$

now $\ell = g_0(c) \left( \frac{1 - \sigma}{1 - \gamma} \right)$

$$\pi(c) = \left[ g_0(c) \left( \frac{1 - \sigma}{1 - \gamma} \right) \right]^\alpha c^\beta$$

$$-g_1 \left( \frac{1 - \sigma}{g_0(c) \left( \frac{1 - \sigma}{1 - \gamma} \right)} \right) \frac{\alpha(1 - \sigma) + \beta(1 - \gamma)}{1 - \sigma}$$

$$-K_1 \frac{\alpha(1 - \sigma)}{1 - \gamma} + \beta = \frac{\alpha(1 - \sigma) + \beta(1 - \gamma)}{1 - \gamma}$$

\[\therefore\] convexity of $\pi(c)$ is determined by sign of
\[
\frac{\alpha(1 - \sigma) + \beta(1 - \gamma)}{1 - r} - 1
\]
\[
= \frac{\alpha(1 - \sigma) + (\beta - 1)(1 - \gamma) - 1 + \gamma}{1 - \gamma}
\]
\[
= \frac{(\gamma - 1)(\beta - 1) + (\sigma - 1)\alpha}{\gamma - 1}
\]
\[
= (\beta - 1) + \left(\frac{\sigma - 1}{\gamma - 1}\right)\alpha.
\]

Observe if \(\alpha + \beta \gg 1\) then both \(\pi(c)\) and \(W(\ell)\) can be convex in their arguments. With CRS one must be concave, the other convex.

Linearity arises when we have \(\gamma = \sigma\) and \(\alpha + \beta = 1\)

\(\gamma\) big relative to \(\sigma\) (scarcity of labor at top)

\(\alpha, \beta\) big - capital productive - we get convexity at top of distribution. Suppose we invoke full employment conditions for capital:
\[
\frac{\alpha(1 - \sigma) + \beta(1 - \gamma)}{1 - \rho} - 1
= \frac{\alpha(1 - \sigma) + (\beta - 1)(1 - \gamma) - 1 + \gamma}{1 - \gamma}
= \frac{(\gamma - 1)(\beta - 1) + (\sigma - 1)\alpha}{\gamma - 1}
= (\beta - 1) + \left(\frac{\sigma - 1}{\gamma - 1}\right)\alpha.
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\[ N_\ell > N_c \quad \pi(1) \geq \pi_R \]

need to check this.

At lowest level of employment, we have (from \( c(\ell) \) function)

\[
1 = \left[ \frac{N_\ell d(\sigma - 1)}{N_c h(\gamma - 1)} \right] \frac{1}{1 - \sigma} \frac{1 - \gamma}{(\ell^*)^{1 - \sigma}}
\]

\[
\therefore \ell^* = \left[ \frac{N_\ell d(\sigma - 1)}{N_c h(\gamma - 1)} \right] \frac{1}{\gamma - 1}
\]

\[
W(\ell^*) = W_R
\]

\[
\therefore c_1 = W_R - \frac{\alpha(1 - \sigma)}{\alpha(1 - \sigma) + \beta(1 - \gamma)}
\]
\[
\left[ \frac{N_\ell d(\sigma - 1)}{N_c h(\gamma - 1)} \right] \frac{\beta}{1 - \sigma} (\ell^*) \frac{\alpha(1 - \sigma) + \beta(1 - \gamma)}{1 - \sigma}.
\]

\(\pi(c)\) defined residually. (Need to check \(\pi(1) > \pi_R\)).

Distribution of earnings: (generated from distribution of endowments by the pricing function).

Look at distribution of translated earnings.

\[(W(\ell) - c_1) \sim (W - c_1)^{-[1-\frac{(\gamma - 1)(\alpha - 1)}{\alpha(\sigma - 1) + \beta(\gamma - 1)}]}\]

distribution of raw skills \(\ell^{-\gamma}\). One way to measure inequality is gain over \(\gamma\) is

\[
1 + \frac{(\gamma - 1)(\sigma - 1)}{\alpha(\sigma - 1) + \beta(\gamma - 1)} > \gamma
\]

i.e.

\[
\frac{1}{\alpha + \beta \frac{\gamma - 1}{\sigma - 1}} > 1
\]

\(\therefore\) more unequal higher returns to scale and higher is \(\sigma\)

(inequality in capital).
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Hedonic Functions (Tinbergen, 1951, 1956 Rosen, 1974).

What can you estimate when you regress $W$ on $\ell$? Obviously we can estimate $c_1$,
\[
\frac{\alpha(\sigma - 1) + \beta(\gamma - 1)}{(\sigma - 1)}
\]
and slope coefficient.

Do not recover any single parameter of interest. We get lowest $\ell$ in market and from distribution of $\ell$ and $c$. We can get $\gamma$, $\sigma$, $h$ (if $c$ fully employed). If we know $\ell^*$, we can get $d$. Know $N_\ell$ and $N_c$ but $\alpha$, $\beta$ in data. Idea (Rosen, 1974) assume perfect data // no error term in model, no omitted variables. Use FOC for firm,
\[
\ln \alpha + (\alpha - 1)\ln \ell + \beta \ln c = \ln W'(\ell)
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i.e \[
\ln \ell = -\frac{\ln \alpha}{\alpha - 1} + \frac{\ln W'(\ell)}{\alpha - 1} - \frac{\beta \ln c}{\alpha - 1}.
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Apparently, we can regress $\ln \ell$ on $\ln W'(\ell)$ but
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Apparently, we can regress $\ln l$ on $\ln W'(l)$ but . . .
\[ \ln c = m_0 + \left( \frac{1 - \gamma}{1 - \sigma} \right) \ln c. \]

We get no independent variation. \( \ln W'(\ell) \), \( \ln c \) and \( \ln \ell \) explain each other perfectly (method wrong).

More general principle:

\[ \text{FOC: } \frac{\partial^2 F}{\partial \ell^2} d\ell + \frac{\partial^2 F}{\partial \ell \partial c} dc = dW'(\ell) \]

\[ d\ell = \frac{\frac{1}{\partial^2 F}}{\left( \frac{\partial^2 F}{\partial \ell^2} \right)} d[W'(\ell)] - \frac{\partial \ell \partial c}{\partial^2 F} dc. \]

Functional dependence between \( c \) and \( W'(\ell) \) does not necessarily imply linear dependence

\[ \therefore \text{we might be able to identify the model. Need shifter in regression. Functional dependence } \not\Rightarrow \text{ linear independence} \]

\[ y = \alpha_0 + \alpha_1 X + \alpha_2 X^2. \]

Obviously \( X \) and \( X^2 \) only dependent but not linearly dependent.
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