Consider Application of I.V. Methods to Models With Heterogeneous Responses

1. a. Linear IV breaks down in the general case for many standard parameters when there is heterogeneity on which people act (A Nonconstant MTE Function)
   3 sets of I.V. conditions in literature
   Neither set implies the other and they identify different treatment parameters

b. More basic question than conventional IV question about efficient weighting. The logically prior question is
   “What Parameter Should We Be Estimating?”
   This is a question in economics and not in statistics. Given the policy question or the intervention, the choice of the instrument is natural.
c. New Ideas: (1) Estimate the MTE using Local Instrumental Variables (LIV) and identify or bound all of the treatment parameters. Can be weighted in different ways. Possess a certain invariance property like familiar policy invariant parameters of traditional econometrics. Or (2) Use policy weighted IV to answer a particular policy question.
Two types of instrumental variable assumptions in the literature

(a) those associated with conventional instrumental assumptions which are implied by the “no selection on heterogeneous gain”

and

(b) those which permit selection on heterogeneous gains $\iff$ nonparametric selection models

(c) both seek conditions under which to justify the standard Linear IV estimator.
In general case, standard Linear IV method breaks down. Not enough for

\[ Z \perp (U_1, U_0), \ D \not\perp Z \]

Must supplement conditions

Two major supplements in Literature
1. (a) Heckman and Robb (1985, 1986)

(b) Imbens and Angrist (1994).

Estimate Different Parameters.
New Idea: A Different Type of Instrumental Variable Estimator That is Locally Adapted. Local Instrumental Variables Estimator: LIV

LIV: Estimates MTE under (i)-(v) and standard regularity conditions for kernel regression. Can be used to estimate, or bound, all parameters, depending on support of $P$.

Another New Idea

Policy weight linear instrumental variable that uses a particular Linear IV estimator to answer a particular policy question.
Common Treatment Effect

\[ Y_1 - Y_0 = \Delta \]

constant \( \Delta \in \mathcal{R} \)

\[ Y_0 = \mu_0(X) + U \]

\[ Y_1 = \mu_1(X) + U \]

\[ Y = \mu + D\Delta + U \]
Common Coefficient Model

\[ E(\Delta \mid X, U_D) \]

a constant \( \Delta \) is degenerate random variable

\[ MTE = ATE = TT = LATE = \Delta \]

If

\[ E(U \mid Z) = 0 \text{ and } \text{Cov}(Z, D) \neq 0 \]

\[ \frac{\text{Cov}(Z, Y)}{\text{Cov}(Z, D)} = \Delta \]
Heterogenous Response Case:

\[ Y_0 = \mu_0 + U_0 \]

\[ Y_1 = \mu_1 + U_1 \]

\[ Y = Y_0 + D(Y_1 - Y_0) \]

\[ = \mu_0 + D(\mu_1 - \mu_0 + U_1 - U_0) + U_0 \]

\[ = \mu_0 + D\Delta + U_0 \]
Random Coefficient Model

2 Cases

(a) \( D \perp \Delta \mid X \implies E(\Delta \mid X, U_D) = E(\Delta \mid X) \)

\( \implies MTE = ATE = TT = LATE \)

Constant MTE Function

(b) \( D \perp \perp \Delta \mid X \implies E(\Delta \mid X, U_D) \neq E(\Delta \mid X) \)

Nonconstant MTE Function

\[
\frac{\text{Cov}(Z, Y)}{\text{Cov}(Z, D)} = \frac{\text{Cov}(Z, D\Delta)}{\text{Cov}(Z, D)} = \mu_1(x) - \mu_0(x) + \frac{\text{Cov}(Z, D(U_1 - U_0))}{\text{Cov}(Z, D)}
\]
Knowledge of \((Z, D)\) and \((Z, (U_0, U_1))\) dependencies not enough.

Need to know joint \((Z, D, U_0, U_1)\) dependencies.

Sufficient for (a):

**Strong Information Condition**

\[
(*) \quad \Pr(D=1 \mid Z, X, U_1-U_0) = \Pr(D=1 \mid Z, X)
\]

\[
\implies
\]

\[
(**) \quad E[U_1 - U_0 \mid Z, X, D = 1] = L(X)
\]

(**) is generically necessary and sufficient for Linear IV to Identify TT and ATE.
(b) violates (**)
Instrumental Variables and ATE/TT

Standard Conditions

(IV-1) $\text{Cov}(Z, D|X) \neq 0$

(IV-2) $U_1, U_0$ mean independent of $Z$ conditional on $X$

More General Condition

(IV-3) $\text{Pr}(D = 1 | X, Z)$ nontrivial function of $Z$.

Conditions are not sufficient for IV to identify ATE or TT in the General Case.
\[\Delta(X, \mathcal{I}) = \mu_1(X) - \mu_0(X) + \mathbb{E}(U_1 - U_0 | X, \mathcal{I})\]

\[\mathcal{I} = (X, D = 1) \implies \]

\[\mu_1(X) - \mu_0(X) + \mathbb{E}(U_1 - U_0 | X, D = 1) = \text{TT}(X)\]

\[\mathcal{I} = (X) \implies \]

\[\mu_1(X) - \mu_0(X) + \mathbb{E}(U_1 - U_0 | X) = \mu_1(X) - \mu_0(X) = \text{ATE}(X).\]
\[ Y = \beta_0(x) + \beta_1(x)D + V \]

\[ \beta_1(x) = \mu_1(x) - \mu_0(x) + E(U_1 - U_0 | X = x, I) = \Delta(x, I) \]

\[ \beta_0(x) = \mu_0 + E[D(U_1 - U_0 - E(U_1 - U_0 | X = x, I)) | X = x] \]

\[ V = (U_0 + D[(U_1 - U_0) - E(U_1 - U_0 | X, I)]) - E[D(U_1 - U_0 - E(U_1 - U_0 | X, I)) | X] \]
Linear IV: Conditions

\[
\frac{\text{Cov}(Z, Y | X)}{\text{Cov}(Z, D | X)} = \beta_1 + \frac{\text{Cov}(Z, V | X)}{\text{Cov}(Z, D | X)}
\]
Conventional Looking Definition:

$Z$ is an instrument for $\Delta(X, I)$ if
\[
\text{Cov}(Z, D|X) \neq 0 \text{ and } E(V|X, Z) = 0
\]
\[
\implies E \left[ U_0 + D(U_1 - U_0 - E(U_1 - U_0|X, I)) \right] |X, Z
\]
\[
= E \left[ D(U_1 - U_0 - E(U_1 - U_0|X, I)) \right] |X
\]

Given (IV-2) condition simplifies

(IV-4) $D(U_1 - U_0 - E(U_1 - U_0|X, I))$ mean independent of $Z$ conditional on $X$.

This is a new condition, necessary and
sufficient for identifying $ATE$ and $TT$.

$IV-1, IV-2, IV-3 \not\equiv IV-4$
ATE Version of IV-4

(ATE-IV) \( D(U_1 - U_0) \) mean independent of \( Z \) conditional on \( X \)

\[ \iff \]

\[ E(D(U_1 - U_0) \mid X, Z) = E(D(U_1 - U_0) \mid X) \]

\[ \implies \]

\[ \Pr(D = 1 \mid X, Z) E(U_1 - U_0 \mid X, Z, D = 1) = \Pr(D = 1 \mid X) E(U_1 - U_0 \mid X, D = 1) \]

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TT Version

\((TT-IV)\) \(U_1 - U_0\) mean independent of \(Z\) conditional on \(D = 1, X\).

Summary

Neither necessary nor sufficient for

\(Z \perp (U_1, U_0)\) for \(Z\) to be a valid instrument.
Example: Generalized Roy Model

\[ D = 1[Y_1 - Y_0 \geq C(Z) + U_2] \]

\[ = 1[U_1 - U_0 \geq C(Z) + U_2 - (\mu_1(X) - \mu_0(X))] \]

\[(X, Z) \perp\perp (U_0, U_1, U_2) \mid X \]

\[ \text{Cov}(Z, D) \neq 0 \]

If \( U_1 - U_0 \equiv 0 \)

\[ \frac{\text{Cov}(Z, Y \mid X)}{\text{Cov}(Z, D \mid X)} = \frac{\text{Cov}(Z, D \mid X) \Delta(X)}{\text{Cov}(Z, D \mid X)} = \frac{\Delta(X)}{\Delta(X)} = 1 \]
If $U_1 - U_0 \neq 0,$

1. Necessary and Sufficient Conditions For Linear IV to Weight MTE The Same As TT:

   \[ \text{Support}(P(Z)) = \{0, p\} \text{ For Some } p \]

2. Necessary and Sufficient Conditions for Linear IV to Weight MTE the Same as ATE

   \[ \text{Support}(P(Z)) = \{0, 1\}. \]
Generalized Roy Model with Imperfect Forsight

\( \mathcal{I} \) denote the information set available to the agent

\[
E(Y_j|\mathcal{I}) = \mu_j(X) + E(U_j|\mathcal{I})
\]

for \( j = 0, 1 \)

\[
E(U_0|\mathcal{I}) = E(U_1|\mathcal{I}) = \theta
\]
Decision rule:

\[ D = 1[ E(Y_1 - Y_0 \mid I) \geq C(Z) + U_2 ] \]

\[ = 1[ \mu_1(X) - \mu_0(X) \geq C(Z) + U_2 ] \]

where

\[ Z \perp \perp (U_0, U_1, U_2) \mid X \]

\[ U_D \perp U_1 - U_0 \mid X \]

\[ (U_1, U_0) \perp Z \mid X \]

\[ U_1 - U_0 \perp D \mid Z, X \]

(**) holds (weak sufficient condition)

**Both ATE and TT Conditions Satisfied.**
If, however,

\[ E(U_0|I) = \alpha_0 \theta \]

\[ E(U_1|I) = \alpha_1 \theta. \]

\[ D = 1 \left[ E(Y_1 - Y_0|I) \geq C(Z) + U_2 \right] \]

\[ = 1 \left[ \mu_1(X) - \mu_0(X) + (\alpha_1 - \alpha_0)\theta \geq C(Z) + U_2 \right] \]

\[ U_1 - U_0 \perp \perp D \perp \perp Z, X \]
Alternative Conditions For Linear IV $\iff$
Semiparametric Selection Based on Index Model
(Imbens-Angrist)

I.A. Counterfactual Conditions

$\mathcal{Z}$ support of $Z$. For each $z \in \mathcal{Z}$, $D_z$ is counterfactual choice that would have been observed if $Z$ had been externally set to $z$.

$D = D_Z$.

(L-1) For each $z \in \mathcal{Z}$, $(U_1, U_0, D_z)$ is independent of $Z$ conditional on $X$.

(L-2) $P(z, x) = \Pr[D = 1 \mid Z = z, X = x]$ is a nontrivial function of $z$.

(L-3) Monotonicity: For any $z, z'$ in the support...
of $Z$, either $D_z \geq D_{z'}$ everywhere (for all individuals) or $D_z \leq D_{z'}$ everywhere (for all individuals).
For $\mathcal{Z} = \{z, z'\}$

Linear IV (Wald Estimator)

$$= \frac{E(Y \mid Z = z, X = x) - E(Y \mid Z = z', X = x)}{\Pr(D = 1 \mid Z = z, X = x) - \Pr(D = 1 \mid Z = z', X = x)}$$

$$= \text{LATE}(Z = z, Z = z', X = x)$$

(L-1) and (L-2) not enough.
In Generalized Roy Model

\[ D = 1[Y_1 - Y_0 > C(Z) + U_2] \]

\[ = 1[U_1 - U_0 > C(Z) + U_2 - (\mu_1(X) - \mu_0(X))] \]

\[ (X, Z) \perp\perp (U_0, U_1, U_2) \mid X \]

\[ \text{Cov}(Z, D) \neq 0. \]

For general \( Z \), ATE and TT fail.
\[ \text{LATE} = E(Y_1 - Y_0 | X = x, D_z = 1, D_{z'} = 0) \]

\[ = \mu_1(x) - \mu_0(x) \]

\[ + E(U_1 - U_0 | X = x, C(z) + U_2 \geq \mu_1(x) - \mu_0(x) + U_1 - U_0 > C(z') + U_2) \]
Comparing the Alternative Conditions

(L-1), (L-2) and (L-3) $\implies$ (IV-1) and (IV-2) $\nRightarrow$

$(ATE - IV) or (TT - IV)$

(IV-1), (IV-2) and (ATE-IV) or (TT-IV) $\nRightarrow$

$(L - 1), (L - 2) and (L - 3)$. 
Example

\( (ATE2-IV) \not\Rightarrow LATE \) identified.

\((ATE2 - IV) \) identifies \( E(Y_1|D_z = 1) \),

\( E(Y_1|D_z = 0) \) but not \( E(Y_1|D_z = 1, D_{z'} = 0) \).

\((ATE2-IV) \) with monotonicity \( \implies \)

\( \Pr[D_z = 1, D_{z'} = 0] E(U_1 - U_0 \mid D_z = 1, D_{z'} = 0) = 0. \)
(TT-IA) \( \not \Rightarrow \) LAT Eidentified.

(TT-IA) identifies \( E(Y_1|D_z = 1) \) and

\[
E(Y_0|D_z = 1) \text{ not } E(Y_0|D_z = 1, D_{z'} = 0) \text{ or }
\]

\[
E(Y_1|D_z = 1, D_{z'} = 0) \text{ for any } (z, z').
\]
\( \text{TT-IA} \Longleftrightarrow \)

\[
(\Pr[D_{z'} = 1|D_z = 1] - \Pr[D_z = 1|D_{z'} = 1]) \\
\]

\[
E(U_1 - U_0|D_z = 1, D'_{z} = 1) \\
\]

\[
= \Pr[D_z = 0|D_{z'} = 1]E(U_1 - U_0|D_z = 0, D'_{z} = 1) - \Pr[D_{z'} = 0|D_z = 1]E(U_1 - U_0|D_z = 1, D'_{z} = 0). \\
\]

With monotonicity
\[ \Pr[D_{z'} = 0|D_z = 1]E(U_1 - U_0|D_z = 1, D_{z'} = 1) \]

\[ = \Pr[D_{z'} = 0|D_z = 1]E(U_1 - U_0|D_z = 1, D_{z'} = 0), \]

so

\[ E(U_1 - U_0|D_z = 1, D_{z'} = 1) \]

\[ = E(U_1 - U_0|D_z = 1, D_{z'} = 0) \]

(TT-IV with monotonicity)
Unsatisfactory State of Affairs

Different Assumptions For Different Parameters

Treatment Effect Literature Is Balkanized