Accounting For Heterogeneity, Diversity and General Equilibrium
In Evaluating Social Programs

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I. Alternative Criteria for Evaluating Social Programs

Outcome for person $i$ in the presence of policy $j = Y_{ji}$

Personal preferences for outcome vector $Y = U_i(Y)$

$Y_{ji} =$ the flow of resources to $i$ under policy $j$.  

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Criteria

(I-1) \( W(j) = W(U_1(Y_{j1}), ..., U_N(Y_{jN})) \).

(I-2) \( B(j) = \sum_{i=1}^{N} U_i(Y_{ji}) \)

(I-3) \( CB(j) = \sum_{i=1}^{N} Y_{ji} \)

(I-4) \( PB(j | j, k) = \frac{1}{N} \sum_{i=1}^{N} 1(U_i(Y_{ji})) \geq U_i(Y_{ki})) \)

(I-5) \( F(\Delta_{jk} | Y_k = y_k, y_k \leq y) \)
(I-6) $\Pr (\Delta_{jk} > 0|Y_k \leq y)\\
Y_{ji} \geq y \quad \text{for } i \in S,\\
Y_{ji} \geq Y_{ki} \quad \text{for } i \in S.\\
I_i = \text{the information set available to agent } i,$
Adding Uncertainty

\((Y_j, Y_k)\) as perceived by agent \(i\).

\[E(U_i(Y_j) | I_i) > E(U_i(Y_k) | I_i).\]

Proportion of people who prefer \(j\) is

\[(I-7) PB(j|j,k) = \int 1(E(U(Y_j|\theta)|I) > E(U(Y_k|\theta)|I))dF(\theta,I).\]

If

\[I_i = (Y_{ji}, Y_{ki}),\] so there is no uncertainty about \(Y_j\) and \(Y_k\),

\[(I-8) PB(j|j,k) = \int 1(U(y_j;\theta) > U(y_k;\theta))dF(\theta,y_j,y_k).\]
Ex post “satisfaction’:

(I-9) \( U_i(Y_{ji}) > E(U_i(Y_{ki}) \mid I_{it}) \).
II. The Data Needed to Evaluate the Welfare State

(II-1) \((Y_{ji}, Y_{ki})\) 1 = 1, ..., I.

Domain of Treatment Effect Literature

Eligible person \(i\) in regime \(j\) has two potential outcomes: \((Y_{ji}^0, Y_{ji}^1)\). A crucial identifying assumption

(A-1) \(Y_{ji}^0 = Y_{0i}\).

Thus we can evaluate \(j\) vs. \(k\) for \(k = 0\).
III. What Can Be Learned From Micro Data and Social Experiments?

We observe $$(Y^0_i, Y^1_i)$$

Cannot form

$$\Delta_i = Y^1_i - Y^0_i$$ for anyone.

$$D_i = 1$$ if person $i$ is a direct participant,

$$D_i = 0$$ if person $i$ is not a direct participant.
$$Y_i = D_i Y_{i1} + (1 - D_i) Y_{i0}$$

The potential outcomes

(III-1) $Y_{i0} = \mu_0 + \varepsilon_{0i}$

(III-2) $Y_{i1} = \mu_1 + \varepsilon_{1i}$

$$E(\varepsilon_0) = E(\varepsilon_1) = 0.$$ 

(III-3) $Y_1 = \mu_0 + (\mu_1 - \mu_0 + \varepsilon_{1i} - \varepsilon_{0i}) D_i + \varepsilon_{0i}$

$$E(Y_1 - Y^0)$$

$$E(Y_1 - Y^0 \mid D = 1)$$
Two Parameters

(C-1): $\varepsilon_{1i} = \varepsilon_{0i}$ so $\Delta_i = \Delta$

(No response heterogeneity given $X$)

or

(C-2): $E(\varepsilon_{1i} - \varepsilon_{0i} \mid D_i = 1) = 0$

(Agents do not enter the program based on gains from it).

(C-3): $E(\varepsilon_{1i} - \varepsilon_{0i} \mid D_i = 1) \neq 0$
Three Regression Models:

(A) \( Y_i = \alpha_0 + \alpha_1 D_i + U_i, \quad E(U_i) = 0. \)

(B) \( Y_i = \alpha_0 + \alpha_1_i D_i + U_i, \quad E(U_i) = 0 \)

where

\[ E(\alpha_{1i}) = \mu_1 - \mu_0 \]

but \( V_i = \alpha_{1i} - E(\alpha_{1i}) = \varepsilon_{1i} - \varepsilon_{0i} \)

satisfies \( E(V_i \mid D_i = 1) = 0 \)

equivalently \( E(\varepsilon_{1i} - \varepsilon_{0i} \mid D_i = 1) = 0. \)
(C) $Y_i = \alpha_0 + \alpha_1 D_i + U_i, \quad E(U_i) = 0$

$$E(V_i \mid D_i = 1) \neq 0 \neq E(\varepsilon_{1i} - \varepsilon_{0i} \mid D = 1).$$
Know

\[ F(y^1 \mid D = 1) \text{ and } F(y^0 \mid D = 1) \]

Don’t Know

\[ F(y^0, y^1 \mid D = 1) \]
Information From Revealed Preference

(III-4) 

\[ D = 1(Y^1 \geq Y^0), \]

The Problem of Recovering Joint Distributions

Treatment Outcome: \( F(y^1|D = 1) \)

\[ Y^1 \sim \begin{pmatrix} Y^1_{(1)} \\ \vdots \\ Y^1_{(N)} \end{pmatrix} \]

Non-Treatment Outcome: \( F(y^0|D = 1) \)

\[ Y^0 \sim \begin{pmatrix} Y^0_{(1)} \\ \vdots \\ Y^0_{(N)} \end{pmatrix} \]

\[ \Delta \sim Y^1 - \prod_{\ell} Y^0 \quad \ell = 1, \ldots, N! \]
IV. Evidence on Impact Heterogeneity and the Value of Self-Assessments and Revealed Preference Information

Evidence on Heterogeneity
Assuming the Gain Is Independent of the Base

\[ R = 1 \text{ if Randomized In; } R = 0 \text{ if Randomized Out} \]

\[ Y = RY^1 + (1 - R)Y^0 = \alpha_0 + \alpha_1R_i + \varepsilon_0 \]

\[ Y_i = \alpha_0 + \bar{\alpha}_1R_i + V_iR_i + \varepsilon_0 \]

where \( E(V_iR_i + \varepsilon_0) = 0 \).
Testing For Ex Ante Stochastic Rationality of Participants

\[(\text{IV-1}) \int_{0}^{\alpha} F_1(y^1|D=1)dy^1 < \int_{0}^{\alpha} F_0(y^0|D=1)dy^0\]

for all \(\alpha \in R_+\).

Evidence from Self-Assessments of Program Participants

Summary of the Evidence on Impact

Heterogeneity and Its Consequences
V. Accounting For General Equilibrium and Heterogeneity in Evaluating Human Capital Policies

A Dynamic General Equilibrium Model of Human Capital Accumulation with Heterogeneous Agents

Individuals live for $\bar{a}$ years and retire after

$a_R \leq \bar{a}$ years.

- $K_{at}$: Stock of physical capital at time $t$.
- $H_{at}^S$: Is the stock of human capital at time $t$ of type $S$ at age $a$.
- $C_{at}$: Consumption
- $I_{at}^S$: For each schooling level.
(V-1) \[ V_{at}(H_{at}^S, K_{at}, S) = \]
\[ \max_{C_{at}, I_{at}^S} \frac{C_{at}^\gamma - 1}{\gamma} + \delta V_{a+1,t+1}(H_{a+1,t+1}^S, K_{a+1,t+1}, S) \]

(V-2) \[ K_{a+1,t+1} \leq K_{a,t}(1 + (1 - \tau_k)r_t) \]
\[ + R_t^S H_{at}^S(1 - I_{at}^S) - \tau_l (R_t^S H_{at}^S(1 - I_{at}^S)) - C_{at} \]

(V-3) \[ H_{a+1,t+1}^S = A^S(\theta) I_{at}^{\alpha_S} H_{at}^{\beta_S} + (1 - \sigma^S) H_{at}^S \]

(V-4) \[ \hat{S} = \operatorname{Argmax}_{S} [V^S(\theta) - D^S + \varepsilon^S] \]
\[
F(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t) =
\begin{align*}
a_3 & \left( a_2 \left( a_1 (\bar{H}_t^1)^{\rho_1} + (1 - a_1)(\bar{H}_t^2)^{\rho_1} \frac{\rho_2}{\rho_1} \right) + (1 - a_2) \bar{K}_t^{\rho_2} \right)^{1/\rho_2}.
\end{align*}
\]

HLT estimate that \( \rho_2 = 0 \) but \( \rho_1 = .693 \).
Tax Effects on Human Capital Accumulation
Analyzing Two Tax Reforms

General-Equilibrium Treatment Effects: A Study of Tuition Policy

Conventional Models of Treatment Effects

The treatment effect for person \( i \) is

\[ \Delta_i = Y_i^1 - Y_i^0. \]

Exploring Increases in Tuition Subsidies

in a General-Equilibrium Model

VI. Summary and Conclusions
Appendix A

The Relationship Between The Requirements of Cost-Benefit Analysis and The Information Produced From Social Experiments and The Microeconometric “Treatment Effect” Literature

Policy $j$

$(D_j = 1$, for program participation$)$ or

not $(D_j = 0)$,

program intensity variables $\varphi_j$

defined under policy $j$

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Policy “0” is no intervention

\( Y^1_{ji} \) and \( Y^0_{ji} \)

direct participation (\( D_j = 1 \))

direct non-participation (\( D_j = 0 \))

\( c_j(\varphi_j) = \) the social cost of \( \varphi_j \)

\( \varphi_j = 0, c_j(0) = 0. \)

\( N_1(\varphi_j) \) be the number of direct program participants and \( N_0(\varphi_j) \)

\[
N_1(\varphi_j)E(Y^1_j \mid D(\varphi_j) = 1, \varphi_j) + \]
\[
N_0(\varphi_j)E(Y^0_j \mid D(\varphi_j) = 0, \varphi_j) - c(\varphi_j)
\]

\( N_1(\varphi_j) + N_0(\varphi_j) = \bar{N}. \)
The change in output in response to a marginal increase in the policy

\[ M(\varphi_j) = \frac{\partial N_1(\varphi_j)}{\partial \varphi_j} \left[ \frac{E(Y_j^1|D_j(\varphi_j) = 1, \varphi_j)}{-E(Y_j^0|D_j(\varphi_j) = 0, \varphi_j)} \right] \\
+ N_1(\varphi_j) \left[ \frac{\partial E(Y_j^1|D(\varphi_j) = 1, \varphi_j)}{\partial \varphi_j} \right] \\
+ N_0(\varphi_j) \left[ \frac{\partial E(Y_j^0|D(\varphi_j) = 0, \varphi_j)}{\partial \varphi_j} \right] \\
- c'_j(\varphi_j). \]
If marginal program intensity changes under policy regime $j$ have no effect on intra-sector mean output:

(a) $\frac{\partial N_1 (\varphi_j)}{\partial \varphi_j}$
the number of people induced into program $j$ by the change in $\varphi_j$,

(b) $E (Y_j^1|D_j(\varphi_j) = 1, \varphi_j) - E (Y_j^0|D_j(\varphi_j) = 0, \varphi_j)$
the mean output difference between participants and nonparticipants.

(c) $c'_j(\varphi_j)$
the direct social marginal cost of policy $j$ at program intensity level $\varphi_j$.

Nowhere on this list are the parameters that receive the most attention in the micro econometric policy evaluation literature.
(a) $E \left( Y_{j1}^1 - Y_{j0}^0 \mid D_j(\varphi_j) = 1, \varphi_j \right)$
   “the effect of treatment on the treated”
   for persons in regime $j$ at policy intensity $\varphi_j$

(b) $E \left( Y_{j1}^1 - Y_{j0}^0 \mid \varphi_j = \bar{\varphi} \right)$
   where $\varphi_j = \bar{\varphi}$ sets $N_1(\bar{\varphi}) = \bar{N}$. This is
   the effect of universal direct participation
   in program $j$ compared to universal
   nonparticipation in $j$ at level of program
   intensity $\bar{\varphi}$.

(c) $E \left( Y_{j1}^1 - Y_{j0}^0 \mid \varphi_j \right)$
   The effect of randomly selecting someone
   for direct treatment and forcing their
   compliance with this treatment compared
   to their position in the no participation state
   under policy $j$ at program intensity level
   $\varphi_j$. 
Net utility from participating in the program is

\[ U_j = X + k(\varphi_j), \text{ where } k \text{ is monotonic in } \varphi_j \]

\( (Y_j^1, X) \) and \( (Y_j^0, X) \) are \( F(y_j^1, X) \) and \( F(y_j^0, X) \)

Roy model, \( X = Y_j^1 - Y_j^0 \) and \( k = 0 \).

\[ D_j(\varphi_j) = 1(U_j \geq 0) = 1(X \geq -k(\varphi_j)) \]

\[ N_1(\varphi_j) = \bar{N} \Pr (U_j \geq 0) = \bar{N} \int_{-k(\varphi_j)}^{\infty} f(x) \, dx \]

\[ N_0(\varphi_j) = \bar{N} \Pr (U_j < 0) = \bar{N} \int_{-\infty}^{-k(\varphi_j)} f(x) \, dx. \]
Total Output:

\[
\tilde{N} \int_{-\infty}^{\infty} y^1 \int_{-k(\varphi_j)}^{\infty} f (y^1, x | \varphi_j) \, dx \, dy^1 \\
+ \tilde{N} \int_{-\infty}^{\infty} y^0 \int_{-\infty}^{-k(\varphi_j)} f (y^0, x | \varphi_j) \, dx \, dy^0 - c_j (\varphi_j)
\]
\begin{align*}
M(\varphi_j) &= \tilde{N}k'(\varphi_j) f_x(-k(\varphi_j)) \\
&+ \tilde{N} \begin{bmatrix}
E(Y_j^1|D(\varphi_j) = 1, X = -k(\varphi_j), \varphi_j) \\
-E(Y_j^0|D(\varphi_j) = 0, X = -k(\varphi_j), \varphi_j)
\end{bmatrix} \\
&\quad + \int_{-\infty}^{\infty} y^1 \int_{-k(\varphi_j)}^{\infty} \frac{\partial f(y^1, x|\varphi_j)}{\partial \varphi_j} \, dx \, dy^1 \\
&\quad + \int_{-\infty}^{\infty} y^0 \int_{-\infty}^{-k(\varphi_j)} \frac{\partial f(y^0, x|\varphi_j)}{\partial \varphi_j} \, dx \, dy^0 \\
&\quad - c'_j(\varphi_j),
\end{align*}

where \( f_x \), the marginal density of \( X \), is evaluated at

\( X = -k(\varphi_j) \). Marginal entry condition:

\( X = -k(\varphi_j) \).

Effect of treatment on the treated \((X = -k(\varphi_j))\).
\[ E (Y_j^1 | D_j(\varphi_j) = 1, X = -k(\varphi_j), \varphi_j) \]
\[ -E (Y_j^0 | D_j(\varphi_j) = 0, X = -k(\varphi_j), \varphi_j) \]
\[ = E (Y^1 - Y^0 | D(\varphi_j) = 1, X = -k(\varphi_j), \varphi_j). \]
Set \( X = -k(\varphi_j) \), the indifference set for this problem.

\[
E(Y^1_j - Y^0_j \mid D_j(\varphi_j) = 1, X, \varphi_j),
\]

\[
A(\varphi_j) = \left\{ N_1(\varphi_j)E(Y^1_j \mid D_j(\varphi_j) = 1, \varphi_j) + N_0(\varphi_j)E(Y^0_j \mid D_j(\varphi_j) = 0, \varphi_j) \right. \]
\[
\left. -c_j(\varphi_j) - \bar{N}E(Y_0 \mid \varphi_0) \right\}
\]
(AA-1) \( E(Y_j^0 \mid D_j(\varphi_j) = 0, \varphi_j) = E(Y_0 \mid D_j(\varphi_j) = 0, \varphi_0) \)

and

(AA-2) \( E(Y_j^0 \mid D_j(\varphi_j) = 1, \varphi_j) = E(Y_0 \mid D_j(\varphi_j) = 1, \varphi_0) \)

(A-2) \( \Pr(D_j = 1 \mid \varphi_j, \varphi_0) = \Pr(D_j = 1 \mid \varphi_j) \)

\( E(Y^0 \mid \varphi_0) = E(Y_0 \mid D_j(\varphi_j) = 1, \varphi_0) \Pr(D_j(\varphi_j) = 1 \mid \varphi_j) \)

\( + E(Y_0 \mid D_j(\varphi_j) = 0, \varphi_0) \Pr(D_j(\varphi_j) = 0 \mid \varphi_j) \).
\[E(Y^0 | \varphi_0) = E(Y^0_j | D_j(\varphi_j) = 1, \varphi_j) \Pr(D_j(\varphi_j) = 1 | \varphi_j)\]

\[+ E(Y^0_j | D_j(\varphi_j) = 0, \varphi_j) \Pr(D_j = 0 | \varphi_j).\]

\[\text{(AA-3) } A(\varphi_j) = N(\varphi_j) E(Y^1_j - Y^0_j | D_j(\varphi_j) = 1, \varphi_j) - c_j(\varphi_j).\]